Mathematics: A Second Language

[The universe] cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word.

—Galileo Galilei (1564–1642), Opere Il Saggiatore, p. 171

For many mathematically challenged students, the thought of working a mathematics problem—processing numbers and more numbers—evokes fear, a tightening of the stomach, and overpowering feelings of anxiety. This feeling, commonly described as “math anxiety,” is more formally defined as “a feeling of intense frustration or helplessness about one’s ability to do math” (Platonic Realms 2007). Such anxiety is just one of the factors contributing to many students’ struggle to learn mathematics. U.S. students as a whole wrestle with mathematics, but a closer analysis across subjects and grade levels reveals an unfortunate link between socioeconomics and school achievement, due in part to academic challenges relating to poverty (Bower 2001). However, students also struggle with language barriers resulting from the mismatch of academic language and their home language or the fact that they are primary speakers of languages other than English (Hacker 2007).

No matter the psychological or socioeconomic reasons, poor mathematical ability has serious consequences, and as educators we must address the question of why so many students are failing. One solution may be to create a classroom where fears can be left at the door and immersion in the language of mathematics can occur. This article describes how mathematics can be defined as a second language and the instructional methods that result from this perspective.

DEFINING MATHEMATICS AS A SECOND LANGUAGE
Greece—home to such great mathematicians as Pythagoras, Plato, and Euclid—stood as the mathematical center of the ancient world. Although Greece later fell to the Romans, Greek culture and mathematical language characteristics did not. As Horace, a Roman poet, wrote, “Captured Greece held its ferocious conqueror captive and introduced the arts to rustic Latium” (Varner 2006, p. 282). The most powerful empire could not suppress the mathematical language of the Greeks. Therefore, our first task is to consider mathematics as a language.

Students can clearly comprehend that mathematics is a powerful tool, but many have difficulty comprehending mathematics as a language. Adams (2003, p. 786) notes that Wakefield identified several commonalities between mathematics and language:

- Abstractions (verbal or written symbols representing ideas or images) are used to communicate.
- Symbols and rules are uniform and consistent.
- Expressions are linear and serial.
Table 1

Adaptation of Gunderson’s English Proficiency Scale to Mathematics

<table>
<thead>
<tr>
<th>Proficiency Level</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero-level mathematics</td>
<td>Cannot answer yes/no questions</td>
</tr>
<tr>
<td></td>
<td>Unable to identify and name any objects</td>
</tr>
<tr>
<td></td>
<td>Understands no mathematics</td>
</tr>
<tr>
<td></td>
<td>Often appears withdrawn and afraid</td>
</tr>
<tr>
<td>Very limited mathematics</td>
<td>Responds to simple questions with mostly yes or no or one-word responses</td>
</tr>
<tr>
<td></td>
<td>Speaks in one- to two-word phrases</td>
</tr>
<tr>
<td></td>
<td>Attempts no extended conversation</td>
</tr>
<tr>
<td></td>
<td>Seldom, if ever, initiates conversation</td>
</tr>
<tr>
<td>Limited mathematics</td>
<td>Responds easily to simple questions</td>
</tr>
<tr>
<td></td>
<td>Produces simple sentences</td>
</tr>
<tr>
<td></td>
<td>Has difficulty elaborating when asked</td>
</tr>
<tr>
<td></td>
<td>Occasionally initiates conversation</td>
</tr>
<tr>
<td>Limited fluency in mathematics</td>
<td>Speaks with ease</td>
</tr>
<tr>
<td></td>
<td>Initiates conversation</td>
</tr>
<tr>
<td></td>
<td>Makes errors in more syntactically complex utterances</td>
</tr>
<tr>
<td></td>
<td>Freely and easily switches code</td>
</tr>
</tbody>
</table>

- Understanding increases with practice.
- Success requires memorization of symbols and rules.
- Translations and interpretations are required for novice learners.
- Meaning is influenced by symbol order.
- Communication requires encoding and decoding.
- Intuition, insightfulness, and "speaking with thinking" accompany fluency.
- Experiences from childhood supply the foundation for future development.

According to Adams, “Mathematics is a language that people use to communicate, to solve problems, to engage in recreation, and to create works of art and mechanical tools” (2003, p. 786). Mathematics apparently shares many of the characteristics that define English as a language, and although historically mathematics was rarely considered a language, current views enable us to consider it as such. Today, unless children grow up in homes in which parents speak and model fluent mathematics, it can be legitimately viewed as a “second language” (Adams 2003; Wakefield 1999, 2000).

In today’s increasingly diverse world, teaching language is a timely issue. Therefore, if mathematics is viewed as a second language, why not teach it as one? The methodologies of teaching English as a second language (ESL) may be applied to the methodologies of teaching mathematics as a second language, or MSL (Curtin 2005). Ruddell notes:

ESL is used to designate classes that are immersion classes, and students in these classes generally have various primary languages and varying language levels of English literacy and oral fluency…. Transition programs are programs intended to bridge ESL classrooms to regular classes, and a method often practiced is one of Sheltered Instruction (SI). (2005, p. 193)

Sheltered instruction is viewed by language educators as an effective approach to language instruction for ESL students in transitional classes and includes “rich language interaction, focus on student’s prior knowledge, integrated/collaborative learning, repetition of ideas and concepts, allowance for students to choose which language to use at any given moment and a low-risk environment for second-language use” (Ruddell 2005, p. 203).

If a student evaluates his or her own mathematical language ability and compares it with a classmate’s, he or she may feel a kinship with a student in the traditional ESL classroom. A student may even be able to place himself or herself in Gunderson’s graduated scale of English oral-language proficiency (see Table 1), modified by replacing the word English with mathematics (Ruddell 2005, pp. 194–95). Thus, a student can rate himself or herself.
as a Mathematics Language Learner (MLL), a designation similar to English Language Learner (ELL).

Once a specific mathematical language level is identified, level-appropriate instruction can be introduced. Thus, teaching mathematics as a second language through a variety of ESL and sheltered instruction strategies instead of more traditional methods may result in higher student comprehension and achievement.

DECODING MATHEMATICS AS A SECOND LANGUAGE

Dissecting a passage of text in a language other than one’s native language is a daunting task and requires a strategy. When dissecting mathematical language, readers are faced with the same challenges, whether the mathematics is in the form of an equation or in the form of a word problem. To aid students in accomplishing such tasks, teachers can apply the framework for problem solving created by George Pólya. Pólya’s (1962) four steps are (1) understanding the problem, (2) devising a plan, (3) carrying out the plan, and (4) looking back. The first and last steps seem particularly relevant to our claim that mathematics is a second language and, therefore, should be presented to students through the use of a second language.

The problem that follows, solving for a system of linear equations, offers a perspective on how each step of the solution relates to mathematics as a language. The problem also illustrates aspects of the Process Standards as defined in NCTM’s Principles and Standards for School Mathematics (2000). These Standards—specifically Problem Solving, Communication, and Representation—are intertwined throughout the process. The problem follows:

Solve this system of equations:

\[
\begin{align*}
X + 4Y + 1Z &= 285 \\
2Y + 2Z &= 90.88 \\
Y + 3Z &= 75.68
\end{align*}
\]

From here, students can dissect the problem while simultaneously following a step-by-step analysis according to Pólya.

STEP 1: UNDERSTANDING THE PROBLEM

This beginning step can be related to a novice foreign language learner who has just been handed a paragraph in the newly studied language. Two steps are critical in understanding the passage: knowing the vocabulary and understanding the structure of the language. Consequently, the first step in reading any passage is understanding how to get started.

Many mathematical words take on meanings different from their everyday English meanings. A student’s ability to translate these words into mathematical language is critical for success. After the student has applied the appropriate definitions, the next step is to reread the sentence and notice any words, sounds, or subject-verb structures that provide any contextual clues. This process is similar to decoding a foreign language, and “math learners benefit when equations are considered from a subject and verb approach” (Wakefield 1999, p. 6). The subject is the aspect of the mathematical equation a reader can understand and connect to past knowledge or work in mathematics, and the verb indicates the direction in which the reader must go to solve the problem successfully.

An additional challenge for the mathematics reader at this step is to understand the unique structures of mathematical language. Like English, Spanish, and French, mathematics can be read from left to right; like old Egyptian, it can be read from right to left; like Japanese and Chinese, it can be read vertically. This uniqueness often requires the mathematics reader to sweep visually from right to left as well as, possibly, up and down, diagonally, and left to right (Barton, Heidema, and Jordan 2002) to understand the “text.” In this example, students are asked to read three separate equations from left to right and align the variables and numbers from top to bottom for clarification. This process requires the student to maintain continuous visual sweeps from left to right and from top to bottom:

\[
\begin{align*}
1X + 4Y + 1Z &= 285 \\
0X + 2Y + 2Z &= 90.88 \\
0X + 1Y + 3Z &= 75.68
\end{align*}
\]

Reading a three-variable equation from left to right as well as reading a column of variables from top to bottom cannot be done simultaneously.

To meet the NCTM Representation Standard, students must “select, apply, and translate among mathematical representations to solve problems” (NCTM 2000, p. 67). Some students may be able to comprehend this problem if they think of X as a video game console, Y as video game controllers, and Z as actual video games. Still others may connect best if they think of X as a prom dress, Y as pairs of shoes, and Z as pieces of jewelry. Translating this mathematics problem into the language each student speaks may be the cornerstone to success. Comprehension is achieved when students are able to immerse themselves in the material and create connections to their own lives, a goal that every mathematics teacher sets for his or her students. Edward Thorndike, frequently referred to as the father of educational psychology, defines intelligence as an individual’s ability to make these connections (Bender 1993, p. 17). However, educators have yet to create the ideal environment in which these math-

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emathematical connections can take place. The purpose of this article is to help create this ideal environment.

**STEP 2: DEVISING A PLAN**
To understand any passage, whether in mathematical or any other language, the reader must be able to translate the text. Translation occurs when a reader is able to define the words and successfully decode the relevant information. Adams found that “a student’s ability to recognize and employ the formal definition is key to understanding and applying concepts when reading mathematical text” (2003, p. 787). In the system of equations presented earlier, the student must grasp the concept of elimination as well as feel comfortable working with two different equations simultaneously. Knowing that elimination is key to solving the problem, the student must then be able to discern which variable is most suitable for removal. Students should realize that selecting equations 2 and 3 and variables Y and Z for initial elimination will be the most direct method.

**STEP 3: CARRYING OUT THE PLAN**
Once a student has quickly reviewed the problem and dealt with any language challenges, he or she must “sum up” all the information and discern meaning—that is, discern the problem’s solution. The student must now carry out the devised plan. In choosing equations (2) and (3), variables Y and Z become obvious choices. Of those two, Y becomes a clear choice, because eliminating Z might introduce fractions into the problem.

\[
\begin{align*}
0X + 2Y + 2Z &= 90.88 \\
2(0X + 1Y + 3Z) &= (75.68)2 \\
0X + 2Y + 2Z &= 90.88 \\
0X + 2Y + 6Z &= 151.36
\end{align*}
\]

From this step, the student is asked to subtract downward, thus forming one equation with one variable and removing the other variable completely:

\[ -4Z = -60.48 \]

At this point, the student will divide both sides of the equation by -4 or multiply both sides by -1/4. Arriving at the conclusion that \( Z = 15.12 \), the student will then substitute the value for \( Z \) into equation (2) and conclude that \( Y = 30.32 \). Again, the student is asked to substitute values for \( Y \) and \( Z \) into equation (1) and will realize that \( X = 148.60 \). The final solution set will be of the following form:

\[
\begin{align*}
X &= 148.60 \\
Y &= 30.32 \\
Z &= 15.12
\end{align*}
\]

**STEP 4: LOOKING BACK**
The student solving the problem presented here must take a moment to look back and ask, “What do those numbers mean?” Teachers hope that students will ask themselves this question, but if they do not, teachers must lead them to do so. Looking back on a solved problem “lets students engage in discussions about the problem-solving process to further enhance their reasoning skills and abilities to explain and justify solutions” (Adams 2003, p. 791).

This fourth and final step hinges on another NCTM Process Standard—Communication, defined by NCTM as students’ ability to “communicate their mathematical thinking coherently and clearly to peers, teachers, and others” (NCTM 2000, p. 60). In this example, after students make the substitutions for each variable, it is important to direct them to ask themselves whether their answers make sense. Students show a complete understanding of mathematical concepts when they know whether their answers seem logical or not.

For the stated problem, both positive and negative answers would be acceptable. However, if the problem had been designed around video games or articles of clothing, would a negative answer have made sense? Can a pair of shoes really cost $-40.67? If a student truly comprehends a problem, he or she should be able to communicate these ideas with a listener of any mathematical skill level. Writing is arguably the best way for a student to reflect on a completed task; it “supports learning because it requires students to organize, clarify and reflect on their ideas—all useful processes for making sense of mathematics” (Burns 2004, p. 30). Writing down all the steps, ideas, and solution strategies applied to a problem allows students to step back and reflect on their own processes as well as develop future methods most appropriate to their unique learning styles.

Applying Pólya’s four steps for solving problems is one effective way of teaching mathematics as a second language. The steps are logical and may be familiar to mathematics educators. Other effective ways can be drawn from strategies often used in second-language instruction.

**INSTRUCTIONAL METHODS FOR THE LANGUAGE OF MATHEMATICS**
Students may benefit from sheltered instruction strategies that result from a teaching perspective of mathematics as a second language. Sheltered instruction incorporates three key concepts: small-group collaborative learning; connecting; and immersion.

Small groups are ideal for creating Mathematics as a Second Language (MLS) classrooms. Working through the mathematics as a language concept as a teacher may be important to classroom success, but
fostering students’ process of learning mathematics as a language through sheltered instruction methodologies may further affect students’ abilities to experiment and acquire mathematical knowledge in depth. Allowing students to meet in small groups enables them to speak with one another quietly and formulate their own conclusions. Working with peers allows them to communicate in their first language and also removes the pressure of speaking in front of the entire class. Sheltered instruction calls for a mixture of group members of all speaking levels (remember Gunderson’s scale) and provides an outlet for students to collaborate. Once the small groups adjourn and the class comes back together as a whole, students have a chance to share their group’s ideas through the language of mathematics.

The process of connecting with the language was previously demonstrated through Pólya’s four steps for problem solving. For students to comprehend a mathematical topic, they must be able to make a connection to previously learned materials or ideas; and to provide students with the chance to make a connection, teachers need a basic understanding of the different cultures and homes each student comes from. Many students come from homes in which neither the mother nor the father has a mathematics-related job. Others may come from different cultures in which no great emphasis is placed on the study of mathematics. Hence, it is important for the teacher to relate the student’s world to the vast world of mathematics. Students need to understand that mathematics is relevant in every area of society, not just the mathematics classroom or even school itself. Although most mathematics teachers would acknowledge the concept of connection, thinking of it as immersing the student in the “culture of mathematics” sheds new light on the idea.

Inviting community members who represent a variety of professions to explain how they use mathematics in their jobs could demonstrate to students the various applications of mathematics. Tracing the history of mathematics from the Egyptians to the Babylonians, the Indians, the Greeks, the Chinese, and today’s mathematicians would be a vivid way of relating each culture to the only language that has survived from the beginning of time. Educators should note that the goal of any connection should be to “develop opportunities for students to strengthen their understanding of mathematics terminology and concepts” (Adams 2003, p. 789).

Immersion has been identified by language educators as one of the best ways for students to learn a foreign language. Although mathematical immersion in the classroom should not mean “sink or swim,” the importance of being able to speak mathematics can still be reinforced. Teachers may find that through immersion, students in mathematics class-rooms develop a deeper understanding of mathematics concepts. An MSL classroom, like the traditional ESL classroom, might display illustrations with the corresponding words underneath. Students could be required to have dictionaries that help them understand and translate words. Teachers could speak in the formal language, but, to further understanding, they would explain in the students’ first language. And parents may be encouraged to immerse their children in mathematics at home.

These strategies attempt to aid students in acquiring a language through continuous immersion rather than periodic instruction (Wakefield 1999, p. 4). If immersion is an excellent strategy for learning such languages as English, Spanish, French, and German, then it seems only logical to teach the language of mathematics the same way.

CONCLUSION
The intent of this article was to answer the question, How should we teach mathematics to our students with an emphasis on comprehension? Thinking about mathematics through the lens of ESL instructional methods may provide insight into weaknesses present in current traditional approaches to teaching mathematics. The non-traditional idea of using approaches more common to language instruction offers suggestions for pedagogical reform that may have the potential for promising results. More accurately, the specific question implied here is this: Is there room in mathematics reform for teachers to use language-specific methods for teaching mathematics such as sheltered instruction and reading strategies? (Franz and Hopper 2007). By treating the mathematics classroom as a classroom for language acquisition, many reading strategies find a home.

BIBLIOGRAPHY


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